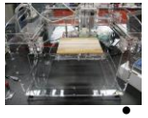
Fundamental Mathematical Model for Direct Write Additive Manufacturing

Robert Strong

Dr. Scott Gold, Dr. Rich Vaia, Dr. Hilmar Koerner

Advantages and disadvantages of direct write technology

writing of material on a 2D or 3D surface from 2D or 3D CAD drawing



Advantages

- Can print 3D objects or on 3D surface
- High resolution applications
- Theoretical wide range of printable materials and temperatures

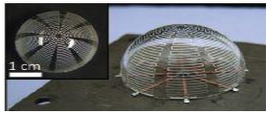
Challenges

- Slow deposition rates
- Limited testing of scale up
- Not commercially available material
- Printing materials based primarily on heavily researched material

Research need

Material and Process Design

- Developed empirically from commonly known systems
- Many models depict droplet spreading, limited bead spreading models
- Current material design does not include input from desired application specifications
- My model allows design of material based from specific application parameters

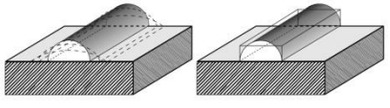


Lewis et al. 2001

Model development

Bead Extrusion

$A = Q/v$



Model Assumptions

$h = \frac{A}{2W}$

- Constant cross sectional area (approx. as rectangle)

$\tau = \eta\dot{\varepsilon}$

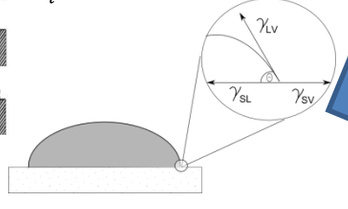
- Newtonian Viscosity Model
- Incompressible Flow

Model development

$\tau(\theta) = \frac{F(\theta)}{2l}$

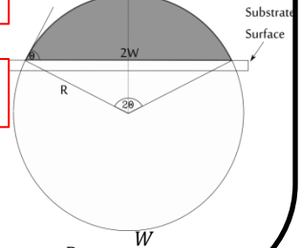
$\frac{F(\theta)}{l} = \gamma_{SV} - \gamma_{SL} - \cos(\theta(t))\gamma_{LV}$

$\frac{dW}{dt} = h/2\,\dot{\varepsilon}$



$$\frac{dW}{dt} = \frac{A}{8W\eta}(\gamma_{SV} - \gamma_{SL} - \cos(\theta(t))\gamma_{LV})$$

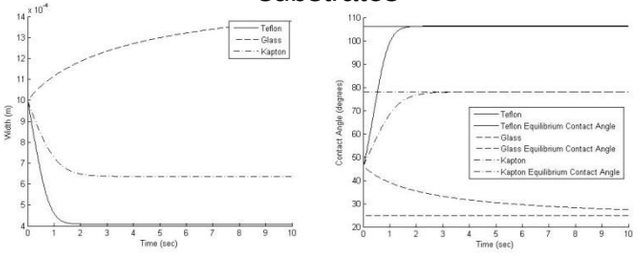
$$A = \frac{R^2}{2}\left(\frac{\pi}{180}2\theta(t) + \sin(2\theta(t))\right)$$

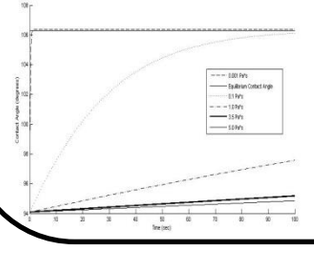


$$R = \frac{W}{\sin(\theta(t))}$$

Model results

Model predicts spreading behavior as expected for water on several different substrates





Model shows viscosity dependence that mirrors what is intuitively expected